Appendix A

Quantifying ternary composition from a ternary graph

Every point in an equilateral triangle uniquely represents a ternary composition $A + B + C = 100\%$. There are several ways to determine the percentage composition in terms of components A, B, C from a ternary graph. Two commonly used methods are illustrated in Figures 1A (Viviani's theorem; left) and 2A (lever rule; right).

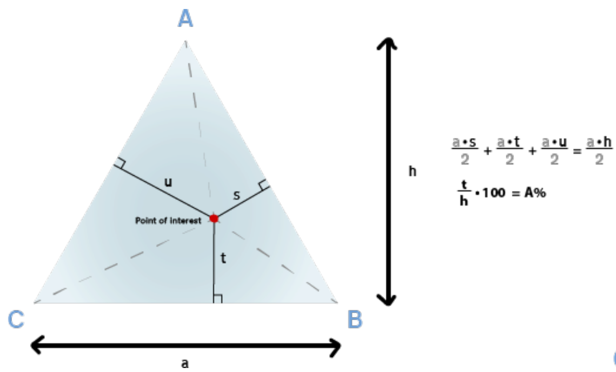

Figure 1A

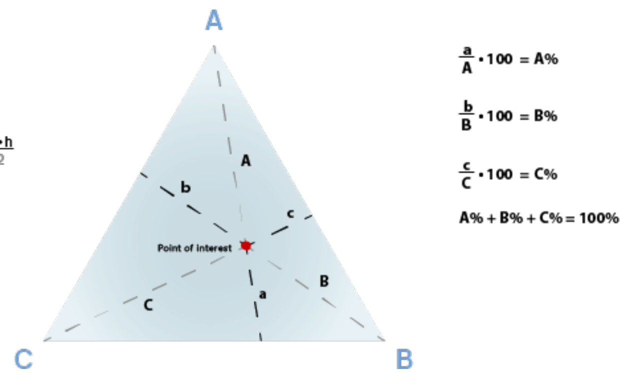

Figure 2A

Graphical interface programming relies on the convention of a 2D Cartesian coordinate system corresponding to pixels on the display, with the origin at the top-left and the y-axis increasingly positive with the pixels moving from top to bottom (this is inverted from the coordinate system common in mathematics). It is therefore necessary to calculate the relative amounts of A, B, C from this representation.

Given a 2D Cartesian coordinate

$$P = \begin{bmatrix} x_p \\ y_p \end{bmatrix} \tag{1}$$

the proportion of A can be determined using the y-axis position relative to the height of the triangle (corresponding to the y-axis position of B, y_b):

$$A\% = 100\% - \frac{y_p}{y_b} \,. \tag{2}$$

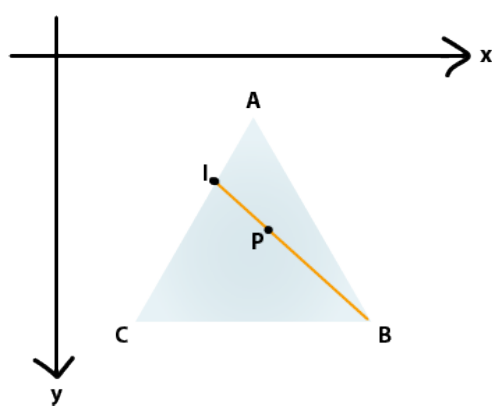

Figure 3A

To find the amount of B, let point I be the intersection of $\overrightarrow{BP}$ and $\overrightarrow{AC}$ (see Figure 3A; above).

It can be shown that

$$I = \begin{bmatrix} \dfrac{(y_b + y_c)x_p}{(x_c - x_a)x_p y_c - (y_p - y_b)(x_c - x_a)} \\ \dfrac{x_i(y_p - y_b)}{x_p} + y_b \end{bmatrix} . \tag{3}$$

The amount of B can be expressed, using the lever rule described by Figure 2A, as

$$B\% = 100\% - \frac{|\overrightarrow{BP}|}{|\overrightarrow{BI}|} . \tag{4}$$

The amount of C is what remains after A and B are removed, hence

$$C\% = 100\% - A\% - B\% . \tag{5}$$